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van den Brink, J.R.

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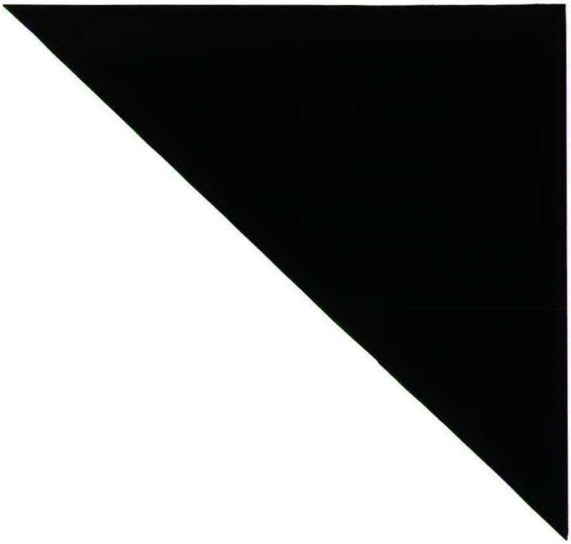

Skewness of the Wage Distribution in a Firm and the Substitutability of Labor Inputs

R. van den Brink

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Skewness of the Wage Distribution in a Firm and the Substitutability of Labor Inputs*

René van den Brink

Department of Econometrics
Tilburg University
P.O. Box 90153
5000 LE Tilburg
The Netherlands

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Abstract

In this paper we present a model of a firm in which the skewness of the wage distribution in the firm depends on the substitutability of the labor inputs. We consider a wage schedule that is based on *games with a permission structure*. We show that for supermodular production technologies this wage schedule satisfies the property that the ratio between the wages of employees in two consecutive levels of a firm lies between one and the *span of control*. Using constant elasticity of substitution (CES) production technologies we show that this ratio increases with the substitutability of the labor inputs. It reaches the upper bound (the span of control) for linear production technologies. It reaches the lower bound (one) for Cobb-Douglas production technologies.

1 Introduction

In this paper we present a model of a firm in which the skewness of the internal wage distribution depends on the substitutability of the labor inputs. We consider a hierarchically structured firm in which a production process takes place according to some supermodular production function. Similarly as in, e.g., Simon (1957), Williamson (1967), Keren and Levhari (1979, 1983), and Radner (1992), the production process is carried out by the workers who constitute the lowest level of the hierarchical firm structure. The participants in higher levels are managers or coordinators who organize and coordinate the production process.

The coordination task of coordinators can have various forms. For example, it can be that the managers in the upper levels of the hierarchical structure make decisions based on information that is available to the workers in the lowest level. The intermediate coordinators process this information from the workers to the upper level managers. In the other direction, the intermediate coordinators process the decisions made by the upper level managers to the workers. If the decision process in the firm is decentralized then also decentralized decisions have to be made by the coordinators. For a survey about these coordination tasks we refer to Radner (1992).

In this paper we consider a hierarchically structured firm to be described by a set of participants (workers and coordinators), a hierarchical coordination structure, and a production technology. We are interested in determining a wage schedule which determines what will be the wage of the workers and the coordinators on the different levels of the coordination structure. We assume this wage schedule to be determined by the *permission value* being a solution concept for *games with a permission structure* as introduced in Gilles, Owen, and van den Brink (1992). The use of this permission value is motivated by the characterizations that are given in van den Brink and Gilles (1996) and van den Brink (1996). We restate these characterizations for the firm model that is described in this paper. It turns out that according to this permission value the wage of a coordinator always is at least as high as the wage of every employee that is directly subordinate to him, while his wage never exceeds the sum of the wages of his direct subordinates. We also show that these bounds are sharp in the sense that there are production technologies for which these bounds are reached. In the special case that the workers are identical in the production process and the firm has constant *span of control*, i.e., every coordinator has the same number of direct subordinates, this means that the ratio between the wage of a coordinator and each of his direct subordinates lies between 1 and the span of control. This lower bound is often assumed in the literature (see, e.g., Simon (1957), Williamson (1967), Calvo and Wellisz (1978, 1979) and Radner (1992), and is supported by, e.g., Carlson (1982). Williamson (1967) argues that also the upper bound is realistic. The model that is discussed in this paper gives a reasoning for these bounds.

After discussing the general model we use *constant elasticity of substitution* (CES) production technologies to illustrate this result. Moreover, we give a reasoning when the ratio between the wages of coordinators and their direct subordinates is close to 1 and when it is close to the span of control s. We show that this ratio is increasing in the elasticity of substitution of the labor inputs. It reaches the lower bound 1 for Cobb-Douglas technologies, and the upper bound s for linear technologies.

The paper is organized as follows. In Section 2 we describe the model and state the main results. In Section 3 we illustrate this model using CES production technologies and show how the skewness of the wage distribution in the firm depends on the substitutability of the labor inputs. Section 4 gives some concluding remarks. Finally, there are two appendices. One which describes games with a permission structure, and the other which gives the proofs of the propositions in Section 2.

2 Hierarchically structured firms and the permission value

In this paper we model a firm as a triple (N, f, S), where the finite set N denotes the set of participants in the firm, f is a production function, and $S: N \to 2^N$ is a *coordination structure*. The participants in $S(i)$ are called the *successors* of participant $i \in N$. They are the participants that are directly subordinate to participant i. On the other hand, the participants in the set $S^{-1}(i) := \{j \in N \mid i \in S(j)\}$, refered to as the *predecessors* of i, are the ones to which i is directly subordinated. By $\widehat{S}$ we denote the *transitive closure* of the coordination structure S, i.e., $j \in \widehat{S}(i)$ if and only if there exists a sequence of participants $(h_1, \ldots, h_t)$ such that $h_1 = i$, $h_{k+1} \in S(h_k)$ for all $1 \leq k \leq t-1$ and $h_t = j$. Thus, $\widehat{S}(i)$ are all direct and indirect subordinates of participant i. We assume the permission structure to have a tree structure, i.e., it satisfies the following two conditions

- there is exactly one participant $i_0 \in N$ such that $S^{-1}(i_0) = \emptyset$ and $\widehat{S}(i_0) = N \setminus \{i_0\}$;
- for every $i \in N \setminus \{i_0\}$ it holds that[1] $|S^{-1}(i)| = 1$ and $S^{-1}(i) \neq \{i\}$.

Similarly as in, e.g., Simon (1957), Williamson (1967), Keren and Levhari (1979, 1983), and Radner (1992), we assume that the only 'productive' participants are the ones in the lowest level of the hierarchy, i.e., the participants that have no successors. We refer to these participants as the *workers* in S and denote this set of participants by $W_S = \{i \in N \mid S(i) = \emptyset\}$. The other participants are the *managers* or *coordinators* who do not actively produce but who coordinate the production process. We denote the set of coordinators in S by $M_S = N \setminus W_S$. For an arbitrary set of participants $E \subset N$ we denote by $\overline{S}(E)$ the set of workers who are (directly or indirectly) subordinate to coordinators in E, i.e., $\overline{S}(E) := \widehat{S}(E) \cap W_S$.

[1] In the sequel we denote by $|E|$ the cardinality of $E \subset N$.

Since the production proces is carried out by the workers in W_S we describe this production proces by a production function $f\colon \mathbb{R}_+^{|W_S|} \to \mathbb{R}$, where $\mathbb{R}_+^{|W_S|}$ is the non-negative orthant of the $|W_S|$-dimensional euclidean space. We assume the production technology to exhibit complementarities with respect to the labor inputs and $f(x) = 0$ if $x_i = 0$ for all $i \in W_S$. This implies the production function to be *supermodular*[2].

Example 2.1 Consider the set of participants $N = \{1, \ldots, 8\}$ and coordination structure

$$S(1) = S(2) = S(3) = S(4) = \emptyset,\ S(5) = \{1,2\},\ S(6) = \{3,4\},\ S(7) = \{6\},\ S(8) = \{5,7\}.$$

This coordination structure is illustrated in Figure 1.

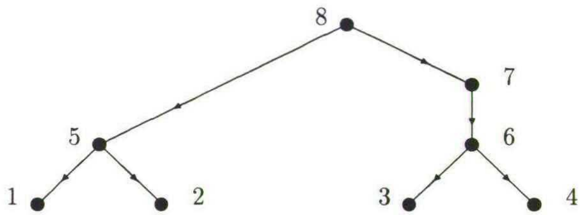

Figure 1: The coordination structure S of Example 2.1

The set of workers is $W_S = \{1,2,3,4\}$, the set of coordinators is the set $M_S = \{5,6,7,8\}$. The set of all subordinates (directly and indirectly) of participant 7 is the set $\widehat{S}(\{7\}) = \{3,4,6\}$. The workers in the set $E = \{2,3,6,8\}$ are the ones in the set $\overline{S}(E) = \{2,3\}$. An example of a production function for this firm is the function $f(x) = (\sum_{i=1}^{4} x_i)^2$ for all $x \in \mathbb{R}_+^4$. In this production function x_i is the labor effort that is provided by worker $i \in W_S$.

[2] A production function $f\colon \mathbb{R}_+^{|W_S|} \to \mathbb{R}$ is *supermodular* if $f(x)+f(y) \leq f(max(x,y))+f(min(x,y))$ for all $x, y \in \mathbb{R}_+^{|W_S|}$, where $max(x,y) \in \mathbb{R}_+^{|W_S|}$ is the vector whose i^{th} component is the maximum of x_i and y_i, and $min(x,y) \in \mathbb{R}_+^{|W_S|}$ is the vector whose i^{th} component is the minimum of x_i and y_i.

A triple (N, f, S) as described above is called a *hierarchically structured firm*. We assume the wages of the participants in a hierarchically structured firm (N, f, S) to be determined by the *permission value* $\varphi(f, S) \in \mathbb{R}^N$ which is studied in van den Brink and Gilles (1996) and van den Brink (1996), and is briefly discussed in Appendix A of this paper. From this appendix it follows that the permission value $\varphi(f, S)$ is given by

$$\varphi_i(f,S) = \sum_{\substack{E \subset W_S \\ (\{i\} \cup \widehat{S}(i)) \cap E \neq \emptyset}} \frac{\Delta_f(E)}{|E \cup \widehat{S}^{-1}(E)|}, \text{ for all } i \in N, \tag{1}$$

where $\widehat{S}^{-1}(E) := \{j \in N \mid \widehat{S}(j) \cap E \neq \emptyset\}$ for all $E \subset N$, and the *dividends* are given by $\Delta_f(E) := \sum_{F \subset E} (-1)^{|E|-|F|} f(x^F)$ with $x^F \in \mathbb{R}_+^{|W_S|}$ being the vector in which all workers in F supply their full effort, and all other workers supply zero[3]. These dividends can be seen as the productivity of the subsets $E \subset W_S$. The dividend $\Delta_f(E)$ represents the production that is generated by E and was not already generated by the subsets of E. For a discussion of these dividends we refer to Harsanyi (1959). The permission value distributes the dividend of a set of workers E equally among the workers in E and their superior coordinators.

Example 2.2 Consider the firm (N, f, S) that is given in Example 2.1. Suppose that $x_i = 1$ if worker $i \in \{1,2,3,4\}$ is active, and $x_i = 0$ otherwise. Thus, $f(x^E) = |E|^2$ for $E \subset W_S$. The dividends $\Delta_f(E)$ for $E \subset W_S = \{1,2,3,4\}$ are given by

$$\Delta_f(E) = \begin{cases} 1 & \text{if } |E| = 1 \\ 2 & \text{if } |E| = 2 \\ 0 & \text{otherwise.} \end{cases}$$

The permission value of participant 1 is given by $\varphi_1(f,S) = \frac{\Delta_f(\{1\})}{|\{1,5,8\}|} + \frac{\Delta_f(\{1,2\})}{|\{1,2,5,8\}|} + \frac{\Delta_f(\{1,3\})}{|\{1,3,5,6,7,8\}|} + \frac{\Delta_f(\{1,4\})}{|\{1,4,5,6,7,8\}|} = \frac{1}{3} + \frac{2}{4} + \frac{2}{6} + \frac{2}{6} = \frac{90}{60}$. In a similar way we can determine the permission values of the other participants. This yields

$$\varphi(f,S) = \frac{1}{60}(90, 90, 79, 79, 150, 134, 134, 204).$$

[3]Note that supermodularity of f here means that $f(x^E) + f(x^F) \leq f(x^{E \cup F}) + f(x^{E \cap F})$ for all $E, F \subset W_S$.

Note that $\sum_{i=1}^{8} \varphi_i(f,S) = 16$ which is exactly equal to the value $f(x^{W_S})$ that can be produced when all workers are active.

A motivation for using this permission value in determining wages is given by the following characterization. It can be verified that the permission value satisfies the following properties. Firstly, it is *efficient*, i.e., for every hierarchically structured firm (N,f,S) it holds that $\sum_{i \in N} \varphi_i(f,S) = f(x^{W_S})$. Secondly, it is *additive*, i.e., for two production functions f, g and coordination structure S it holds that $\varphi(f+g,S) = \varphi(f,S) + \varphi(g,S)$, where $(f+g)\colon \mathbb{R}_+^{|W_S|} \rightarrow \mathbb{R}$ is given by $(f+g)(x) = f(x) + g(x)$. A participant $i \in N$ is *inessential* in a hierarchically structured firm (N,f,S) if every worker that he coordinates does not add anything in the production process, i.e., for every $j \in (\{i\} \cup \widehat{S}(i)) \cap W_S$ and every pair $x, y \in \mathbb{R}^{W_S}$ with $x_h = y_h$ for all $h \in W_S \setminus \{j\}$ it holds that $f(x) = f(y)$. The permission value satisfies the *inessential participant property* meaning that for every inessential participant i it holds that $\varphi_i(f,S) = 0$. In the literature about the firm it is often argued that the wage of a manager in a firm is always at least as high as the wages of any of its subordinates (see, e.g., Simon (1957), Williamson (1967), Calvo and Wellisz (1978, 1979), and Carlson (1982)). The permission value satisfies this property, i.e., $\varphi_i(f,S) \geq \varphi_j(f,S)$ for all $i \in M_S$ and $j \in S(i)$. This property is refered to as *structural monotonicity*. Finally, a worker $i \in W_S$ is called *necessary* in the production process if without his labor effort nothing can be produced, i.e., $f(x) = 0$ if $x_i = 0$. The permission value satisfies the *necessary worker property* meaning that for every necessary worker $i \in W_S$ it holds that $\varphi_i(f,S) \geq \varphi_j(f,S)$ for all $j \in N$. It turns out that the permission value is the unique wage schedule that satisfies these five properties. Therefore, if we want a wage schedule to satisfy these properties it has to be determined by the permission value. (The proof of this result is implicitly given in the proof of the characterization of the disjunctive permission value in van den Brink (1996), and is therefore omitted.)

Next, we compare the permission value of a coordinator in a hierarchicaly structured firm with the permission value of his successors. It turns out that the permission value of a coordinator in a hierarchically structured firm is at least as high as the permission value of any of its successors, and it is at most equal to the sum of the permission values of its successors.

Proposition 2.3 *For every hierarchically structured firm (N, f, S) it holds that*

$$\max_{j \in S(i)} \varphi_j(f, S) \leq \varphi_i(f, S) \leq \sum_{j \in S(i)} \varphi_j(f, S) \text{ for all } i \in M_S,$$

where M_S denotes the set of coordinators in S.

The proof of this proposition can be found in Appendix B. The next proposition shows that these bounds are sharp in the sense that there are hierarchically structured firms such that the inequalities are equalities. The proof of this proposition also can be found in Appendix B.

Proposition 2.4 *For every hierarchically structured firm (N, f, S) and coordinator $i \in M_S$ it holds that*

(i) *if there exists a $j \in S(i)$ such that for every $h \in \overline{S}(i) \setminus \overline{S}(j)$ and every $E \subset W$ with $E \cap \overline{S}(j) = \emptyset$ it holds that $f(x^E) = f(x^{E \setminus \{h\}})$ then $\varphi_i(f, S) = \max_{j \in S(i)} \varphi_j(f, S)$;*

(ii) *if for every $h \in \overline{S}(i)$ and every $E \ni h$ it holds that $f(x^E) = f(x^{E \setminus \{h\}}) + f(x^{\{h\}})$ then $\varphi_i(f, S) = \sum_{j \in S(i)} \varphi_j(f, S)$.*

The first part of this proposition states that the payoff of a coordinator i is equal to the payoff of his highest payed successor j if every worker h that is coordinated by i and not by $j \in S(i)$ does not increase the productivity of any set of workers E that does not contain workers coordinated by j. In other words, the payoff of coordinator i is equal to the payoff of his highest payed successor j if the workers that are coordinated by i but not by j need workers that are coordinated by j in order to be productive. The second part of the proposition states that coordinator i's payoff is equal to the sum of the payoffs of all his successors if every worker h that is coordinated by i increases the productivity of every set of workers by the value $f(x^{\{h\}})$ which he also can produce on his own without cooperating with other workers. In other words, coordinator i's payoff is equal to the sum of the payoffs of all his successors if all workers that are coordinated by i only produce on their own in the sense that there are no synergies when they cooperate with other workers.

In the mentioned literature about the firm it is assumed that the workers in W_S are identical in the production process. With (1) it can be shown that in that case the permission values of participants who take similar positions in the coordination structure are the same. From Proposition 2.4 we can straightforwardly derive that in this case the ratio between the wage of a coordinator i and the wage of each of his successors lies between 1 and the number of successors of i. If we also assume that the hierarchical coordination (tree) structure has *constant span of control*, i.e., every coordinator has the same number of successors s, as is done in, e.g., Simon (1957) and Williamson (1967), then we have the following.

Corollary 2.5 *For every hierarchically structured firm (N, f, S) with constant span of control s and identical workers it holds that*

$$1 \leq \frac{\varphi_i(f,S)}{\varphi_j(f,S)} \leq s \text{ for all } i \in M_S \text{ and } j \in S(i).$$

In the literature it is often assumed that the ratio between the wage of a manager and the wage of his successors lies between 1 and the span of control s. Corollary 2.5 shows that the permission value satisfies this assumption. In the next section we show how for constant elasticity of substitution (CES) production functions this ratio depends on the substitutability of the labor inputs.

3 Constant elasticity of substitution (CES) production technologies

In this section we illustrate the model that is discussed in the previous section using constant elasticity of substitution (CES) production functions. As in the previous section, we assume that the production process that takes place within the firm is carried out by the workers in the lowest level of the hierarchy. The other participants are coordinators who coordinate the production process.

3.1 A 2-level firm

In this subsection we consider a constant elasticity of substitution (CES) production process which produces an output using two types of labor inputs that can be provided by the workers 1 and 2 according to the production function $f_\rho\colon \mathbb{R}^2 \to \mathbb{R}$ given by

$$f_\rho(x) = \gamma\left(\alpha(x_1)^\rho + (1-\alpha)(x_2)^\rho\right)^{\frac{1}{\rho}} \quad \text{for all } x = (x_1, x_2)^T \in \mathbb{R}^2,\ 0 < \rho \leq 1.$$

In this production function $x_i \in \mathbb{R}$ is the amount of labor that is provided by worker $i \in W = \{1, 2\}$. In order to establish symmetry of the workers we take $\alpha = \frac{1}{2}$.

We assume that every worker $i \in W$ can choose either to provide all his labor effort in producing the output (in which case $x_i = 1$) or to provide nothing at all (in which case $x_i = 0$). Thus, if the workers in $E \subset W$ are active in the production process then the labor inputs are given by the vector $x^E \in \mathbb{R}^2$ given by $x_i^E = \begin{cases} 1 & \text{if } i \in E \\ 0 & \text{else.} \end{cases}$
The production technology thus is given by

$$f_\rho(x^E) = \begin{cases} \gamma\left(\frac{1}{2}\right)^{\frac{1}{\rho}} & \text{if } |E| = 1 \\ \gamma & \text{if } |E| = 2. \end{cases}$$

The dividends for this production technology are given by

$$\Delta_{f_\rho}(E) = \begin{cases} \gamma\left(\frac{1}{2}\right)^{\frac{1}{\rho}} & \text{if } |E| = 1 \\ \gamma\left(1 - \left(\frac{1}{2}\right)^{\frac{1}{\rho}-1}\right) & \text{if } |E| = 2 \end{cases}$$

Next, suppose that these production processes take place within a firm with 3 employees and coordination structure S on $N = \{1, 2, 3\}$ given by $S(1) = S(2) = \emptyset,\ S(3) = \{1, 2\}$. This coordination is illustrated in Figure 2. It has two hierarchical levels and span of control $s = 2$.

With (1) it then follows that the permission values for the coordinator 3 and the workers 1 and 2 are given by

$$\varphi_3(f_\rho, S) = \frac{\gamma}{3}\left(1 + \left(\frac{1}{2}\right)^{\frac{1}{\rho}}\right),$$

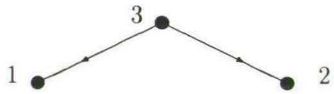

Figure 2: Coordination structure S of the firm in Section 3.1

and

$$\varphi_i(f_\rho, S) = \frac{\gamma}{3}\left(1 - \left(\frac{1}{2}\right)^{\frac{1}{\rho}+1}\right) \text{ for } i \in \{1,2\}.$$

The ratio between the wages of coordinator 3 and the workers 1 and 2, denoted by $\delta(\rho)$, then is given by

$$\delta(\rho) := \frac{\varphi_3(f_\rho, S)}{\varphi_1(f_\rho, S)} = \frac{1 + (1/2)^{\frac{1}{\rho}}}{1 - (1/2)^{\frac{1}{\rho}+1}} \text{ for } \rho \in (0,1].$$

Since $\frac{d\delta(\rho)}{d\rho} = -\frac{2\ln(1/2)(1/2)^{\frac{1}{\rho}+1}}{(\rho(1-(1/2)^{\frac{1}{\rho}+1}))^2} > 0$ for $\rho \in (0,1]$, this ratio is increasing in ρ, i.e., the difference between the wages of the coordinator and the workers increases if the substitutability of the labor inputs increases. (Note that $\delta(\rho)$ does not depend on the parameter γ.)

Below we consider the two extreme cases $\rho = 1$ and $\rho \to 0$, and an intermediate case $\rho = 1/2$.

(i) If $\rho = 1$ we have a linear production technology (as in Williamson (1967)) with production function

$$f_1(x^E) = \frac{\gamma}{2}\left(x_1^E + x_2^E\right) = \frac{\gamma}{2}(|E|) \text{ for all } E \subset W = \{1,2\}.$$

In this case the labor inputs are perfect substitutes. The dividends are given by

$$\Delta_{f_1}(E) = \begin{cases} \frac{\gamma}{2} & \text{if } |E| = 1 \\ 0 & \text{if } |E| = 2 \end{cases}$$

The permission values of the employees then are given by $\varphi_3(f_1, S) = \frac{\gamma}{2}$ and $\varphi_i(f_1, S) = \frac{\gamma}{4}$ for $i \in \{1, 2\}$, and thus $\delta(1) = 2$. Thus, in case of perfect substitutability of the labor inputs the ratio between the wage of the coordinator and the wage of the workers is equal to the span of control $s = 2$.

(ii) If $\rho \to 0$ then the production function approaches a Cobb-Douglas production function given by

$$f_0(x^E) = \gamma x_1^E x_2^E = \begin{cases} \gamma & \text{if } E = W \\ 0 & \text{else.} \end{cases}$$

This production function expresses complementarity of the labor inputs. In this case the dividends are given by

$$\Delta_{f_0}(E) = \begin{cases} 0 & \text{if } |E| = 1 \\ \gamma & \text{if } |E| = 2, \end{cases}$$

and the permission values are given by $\varphi_i(f_0, S) = \frac{\gamma}{3}$ for $i \in \{1, 2, 3\}$, and thus $\delta(0) = 1$. Thus, in this case the wages of the coordinator and the workers are equal, i.e., the ratio between the wage of a coordinator and the wage of the workers is equal to 1.

(iii) Finally, we consider an intermediate case with $\rho = \frac{1}{2}$ in which case the production function is given by

$$f_{1/2}(x^E) = \gamma \left(\frac{1}{2} \left(\sqrt{x_1^E} + \sqrt{x_2^E} \right) \right)^2 = \frac{\gamma}{4}(|E|)^2 \text{ for all } E \subset W.$$

In this case the dividends are given by

$$\Delta_{f_{1/2}}(E) = \begin{cases} \frac{\gamma}{4} & \text{if } |E| = 1 \\ \frac{\gamma}{2} & \text{if } |E| = 2, \end{cases}$$

and the permission values are given by $\varphi_3(f_{1/2}, S) = \frac{5}{12}$, and $\varphi_i(f_{1/2}, S) = \frac{7}{24}$ for $i \in \{1, 2\}$. Thus, $1 < \delta(1/2) = \frac{10}{7} < 2$.

Thus, the ratio between the wage of the coordinator and the wages of the workers in a 2-level firm with CES production technologies and span of control s equal to 2 lies between 1 and the span of control 2. This ratio depends on the character of the production process that is carried out within the firm. The lower bound 1 is reached for linear production technologies. The upper bound $s = 2$ is reached for Cobb-Douglas production technologies.

For a 2-level firm with span of control $s \geq 2$ we can derive similar results. In this case also the wage differential is increasing in ρ with $\delta(1) = s$ for the linear production technology, and $\delta(0) = 1$ for the Cobb-Douglas technology.

3.2 A 3-level firm

In this example we consider a CES production process which produces an output using four types of identical labor inputs that can be provided by the workers 1, 2, 3, and 4 according to the production function $f_\rho \colon \mathbb{R}^4 \to \mathbb{R}$ given by

$$f_\rho(x) = \gamma \left(\sum_{i=1}^{4} \left(\frac{1}{4} x_i \right)^\rho \right)^{\frac{1}{\rho}} \quad \text{for all } x \in \mathbb{R}^4,\ 0 < \rho \leq 1.$$

Similarly as in the previous subsection we assume the workers either to provide full labor effort or to provide nothing. The labor input vector x^E, $E \subset W_S$ is defined similarly as in the previous subsection. These production processes take place within a firm with 7 employees and hierarchical coordination structure S on $N = \{1, \ldots, 7\}$ given by

$$S(1) = S(2) = S(3) = S(4) = \emptyset,\ S(5) = \{1,2\},\ S(6) = \{3,4\},\ S(7) = \{5,6\}.$$

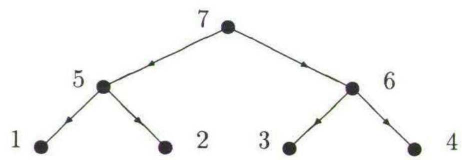

Figure 3: Coordination structure S of the firm in Section 3.2

This coordination structure is illustrated in Figure 3. It has three hierarchical levels and span of control $s = 2$.

Again, we consider the following three special cases with respect to ρ.

(i) In case $\rho = 1$ we have a linear production technology with production function

$$f_1(x^E) = \frac{\gamma}{4}\sum_{i=1}^{4} x_i^E = \frac{\gamma}{4}(|E|) \text{ for all } E \subset W = \{1,2,3,4\}.$$

In this case the dividend $\Delta_{f_1}(x^E)$ is equal to $\frac{\gamma}{4}$ if $|E| = 1$, and equal to 0 otherwise. The permission values are given by $\varphi(f_1, S) = \frac{\gamma}{12}(1,1,1,1,2,2,4)$, and thus, the ratio between the wage of a coordinator and the wage of his successors is equal to the span of control $s = 2$.

(ii) If $\rho \to 0$ then the production function approaches a Cobb-Douglas production function given by

$$f_0(x^E) = \begin{cases} \gamma & \text{if } E = \{1,2,3,4\} \\ 0 & \text{else.} \end{cases}$$

In this case the dividend $\Delta_{f_0}(x^E)$ is equal to γ if $|E| = 4$, and equal to 0 otherwise. The permission values are given by $\varphi_i(f_0, S) = \frac{\gamma}{7}$ for all $i \in N$.

Thus, the wages are the same in all levels of the hierarchical structure, i.e., the ratio between the wage of a coordinator and the wage of his successors is equal to one.

(iii) Finally, we consider an intermediate case with $\rho = \frac{1}{2}$ in which case the production function is given by

$$f_{1/2}(x^E) = \frac{\gamma}{16} \cdot \left(\sum_{i=1}^{4} \sqrt{x_i^E} \right)^2 = \frac{\gamma}{16}(|E|)^2 \text{ for all } E \subset W.$$

In this case the dividends are given by

$$\Delta_{f_{1/2}}(E) = \begin{cases} \frac{\gamma}{16} & \text{if } |E| = 1 \\ \frac{\gamma}{8} & \text{if } |E| = 2, \\ 0 & \text{otherwise,} \end{cases}$$

and the permission values are given by

$$\varphi_i(f_{1/2}, S) = \begin{cases} \frac{49\gamma}{480} & \text{if } i \in \{1,2,3,4\} \\ \frac{83\gamma}{480} & \text{if } i \in \{5,6\} \\ \frac{118\gamma}{480} & \text{if } i = 7. \end{cases}$$

Thus, in this intermediate case it holds that (i) the ratio between the wages of a coordinator and of his successors is larger for the coordinators in the lower level of the hierarchy, and (ii) at every level this ratio is strictly between 1 and the span of control $s = 2$.

Again, similar results can be derived for firms with span of control $s \geq 2$.

4 Concluding remarks

In this paper we have illustrated how games with a permission structure and the permission value can give a reasoning for the assumption that the ratio between the wages of employees in two consecutive levels of a firm lies between 1 and the span of control s. Using CES production technologies we have shown that this ratio depends on the character of the production process that is carried out within the firm. The upper bound s turned out to be reasonable for linear production technologies. The lower bound 1 is reasonable for Cobb-Douglas technologies.

The model that is developed in this paper can be applied in various ways. In van den Brink and Ruys (1996), for example, it is used in endogenously determining the optimal size of the firm. In that paper it is assumed that the span of control of a firm is constant and exogenously given but the number of hierarchical levels variabel. The owner of the firm is represented by the topman i_0 and his profit is given by the permission value of this topman. He then chooses the number of levels (and thus firm size) that maximizes his profit, i.e., that maximizes his permission value. This gives an alternative model to the models of Williamson (1967) and Qian (1994).

Appendix A: Games with a permission structure

In this appendix we briefly discuss *games with a permission structure* which are introduced in Gilles, Owen, and van den Brink (1992) and the permission value which is extensively studied in van den Brink and Gilles (1996) and van den Brink (1996). A game with a permission structure is a triple (N, v, S), where N is a finite set of players, $v: 2^N \to \mathbb{R}$ is a *characteristic function* such that $v(\emptyset) = 0$, and $S: N \to 2^N$ is a *permission structure*. By $\widehat{S}$ we denote the *transitive closure* of the permission structure S, i.e., $j \in \widehat{S}(i)$ if and only if there exists a sequence of players $(h_1, \ldots, h_t)$ such that $h_1 = i$, $h_{k+1} \in S(h_k)$ for all $1 \leq k \leq t-1$ and $h_t = j$. Further we denote $\widehat{S}^{-1}(i) := \{j \in N \mid i \in \widehat{S}(j)\}$. Now we assume that a player $i \in N$ needs permission from all players in $\widehat{S}^{-1}(i)$ before he is allowed to cooperate. Thus, the cooperation possibilities are limited as expressed by the modified characteristic function $\mathcal{R}_S(v): 2^N \to \mathbb{R}$ given by

$$\mathcal{R}_S(v)(E) = v(\sigma(E)),$$

where $\sigma(E) = \{i \in E \mid \widehat{S}^{-1}(i) \subset E\}$ for all $E \subset N$. Now, the *permission value* φ is the allocation rule that assigns to every player i in a game with a permission structure (N, v, S) its *Shapley value* (Shapley (1953)) in $\mathcal{R}_S(v)$, i.e.,

$$\varphi_i(v, S) = Sh_i(\mathcal{R}_S(v)),$$

where

$$Sh_i(v) = \sum_{\substack{E \subset N \\ E \ni i}} \frac{\Delta_v(E)}{|E|}, \text{ for all } i \in N,$$

with *dividends* (see Harsanyi (1959)) are given by $\Delta_v(E) := \sum_{F \subset E} (-1)^{|E|-|F|} v(F)$ for all $E \subset N$.

In Section 2 we represented a firm by a triple (N, f, S) with N the set of participants (consisting of a set of workers W_S, and a set of coordinators M_S), a production function $f\colon \mathbb{R}^{|W_S|} \to \mathbb{R}$, and a permission structure $S\colon N \to 2^N$. The corresponding game with permission structure (N, v, S) is constructed as follows. First we define the characteristic function $\overline{v}$ on the set of workers W_S by $\overline{v}(E) = f(x^E)$ for all $E \subset W_S$, where x^E is the vector of labor inputs that can be provided by the workers in E. We extend this characteristic function to the class N of all participants using the characteristic function v given by $v(E) = \overline{v}(E \cap W_S)$ for all $E \subset N$. Note that the dividends discussed in Section 2 are equal to the dividends discussed here, i.e., $\Delta_f(E) = \Delta_v(E)$ for all $E \subset N$. Then the permission value of hierarchically structured firm (N, f, S) is given by $\varphi(f, S) = \varphi(v, S)$, i.e.,

$$\varphi_i(f, S) = \varphi_i(v, S) = \sum_{\substack{E \subset N \\ E \ni i}} \frac{\Delta_{\mathcal{R}_S(v)}(E)}{|E|}$$

$$= \sum_{\substack{E \subset W_S \\ (\{i\} \cup \widehat{S}(i)) \cap E \neq \emptyset}} \frac{\Delta_v(E)}{|E \cup \widehat{S}^{-1}(E)|} = \sum_{\substack{E \subset W_S \\ (\{i\} \cup \widehat{S}(i)) \cap E \neq \emptyset}} \frac{\Delta_f(E)}{|E \cup \widehat{S}^{-1}(E)|}, \text{ for all } i \in N.$$

(The third equality follows from a result that is stated in Gilles, Owen, and van den Brink (1992).) This is the value that is given in equation (1) and is used in determining wage schedules in hierarchically structured firms.

Appendix B: Proofs

We conclude this paper by giving the proofs of the propositions stated in Section 2. We do this using an alternative expression of the Shapley value, namely

$$Sh_i(v) = \sum_{\substack{E \subset N \\ E \ni i}} \frac{(|N| - |E|)!((|E| - 1)!}{(|N|)!}(v(E) - v(E \setminus \{i\})) \text{ for all } i \in N. \tag{2}$$

Let (N, v, S) be the game with permission structure cooresponding to the hierarchically structured firm (N, f, S) as described in Appendix A. Using expression (2) in determining the permission value for (N, f, S) yields[4]

$$\begin{aligned}
\varphi_i(f, S) &= \varphi_i(v, S) = \sum_{E \ni i} p(E)(\mathcal{R}_S(v)(E) - \mathcal{R}_S(v)(E \setminus \{i\})) \\
&= \sum_{E \ni i} p(E)(v(\sigma(E)) - v(\sigma(E \setminus \{i\})) \\
&= \sum_{E \ni i} \sum_{\substack{F \subseteq \sigma(E) \cap \overline{S}(E) \\ \widehat{S}(i) \cap F \neq \emptyset}} p(E) \left(\overline{v}(F) - \overline{v}(F \setminus \overline{S}(i))\right) \\
&= \sum_{E \ni i} \sum_{\substack{F \subseteq \sigma(E) \cap \overline{S}(E) \\ \widehat{S}(i) \cap F \neq \emptyset}} p(E) \left(f(x^F) - f(x^{F \setminus \overline{S}(i)})\right) \text{ for all } i \in N,
\end{aligned}$$

where $p(E) := \frac{(\#N - |E|)!((|E|-1)!}{(\#N)!}$ and $\overline{S}(E) := \widehat{S}(E) \cap W_S$ for all $E \subset N$.

Proof of Proposition 2.3

Let (N, f, S) be a hierarchically structured firm, and let (N, v, S) be the corresponding game with permission structure as described in Appendix A.

[4]In the remainder of the proof we denote $\sum_{E \ni i}$ for $\sum_{\substack{E \subset N \\ E \ni i}}$.

Since f is supermodular and $f(x) = 0$ if $x_i = 0$ for all $i \in N$ it holds that v is monotone and convex[5], and thus $\mathcal{R}_S(v)$ is monotone and convex (see Gilles, Owen, and van den Brink (1992)), i.e.,

$$\mathcal{R}_S(v)(E) \leq \mathcal{R}_S(v)(F) \text{ for all } E \subset F \subset N,$$

and

$$\mathcal{R}_S(v)(E \cup F) + \mathcal{R}_S(v)(E \cap F) \geq \mathcal{R}_S(v)(E) + \mathcal{R}_S(v)(F) \text{ for all } E, F \subset N.$$

For notational convenience we denote $w = \mathcal{R}_S(v) = v(\sigma(E))$ for all $E \subset N$ in the remainder of the proof. As noted in the text, the first inequality of Proposition 2.3 follows from structural monotonicity of the permission value. This can also be shown with expression (2) using the facts that for every $i \in M_S$ and $j \in S(i)$ monotonicity of w implies that

(i) $w(E) - w(E \setminus \{j\}) = 0$ for all $E \not\ni i,\ E \ni j$ (since $\sigma(E) = \sigma(E \setminus \{j\})$ for all $E \subset N$ with $E \not\ni i$);

(ii) $w(E) - w(E \setminus \{i\}) \geq 0$ for all $E \ni i,\ E \not\ni j$ (since $\sigma(E \setminus \{i\}) \subset \sigma(E)$ for all $E \subset N$);

(iii) $w(E) - w(E \setminus \{j\}) \leq w(E) - w(E \setminus \{i\})$ for all $E \ni i, j$ (since $\sigma(E \setminus \{i\}) \subset \sigma(E \setminus \{j\})$ for all $E \subset N$).

With this it follows that

$$\begin{aligned}\varphi_i(f, S) &= \varphi_i(v, S) = Sh_i(w) = \sum_{E \ni i} p(E)(w(E) - w(E \setminus \{i\})) \\ &= \left(\sum_{E \ni i,j} p(E)(w(E) - w(E \setminus \{i\}))\right) + \left(\sum_{\substack{E \ni i \\ E \not\ni j}} p(E)(w(E) - w(E \setminus \{i\}))\right)\end{aligned}$$

[5] A characteristic function v on N is *monotone* if $v(E) \leq v(F)$ for all $E \subset F \subset N$. A characteristic function v on N is *convex* if $v(E \cup F) + v(E \cap F) \geq v(E) + v(F)$ for all $E, F \subset N$. Note the similarity between convexity of v and supermodularity of f (see footnote 3).

$$
\begin{aligned}
&\geq \sum_{E \ni i,j} p(E)(w(E) - w(E \setminus \{j\})) \\
&= \sum_{E \ni j} p(E)(w(E) - w(E \setminus \{j\})) = Sh_j(w) = \varphi_j(v, S) = \varphi_j(f, S).
\end{aligned}
$$

In order to prove the second inequality we first note that for every $i \in M_S$ with $S(i) = \{j_1, \ldots, j_s\}$ convexity of w implies that for every $k \in \{1, \ldots, s\}$ and $E \subset N$ it holds that

$$
w(E) \geq w(E \setminus \{j_k\}) + w\left(E \setminus \bigcup_{l=k+1}^{s} j_l\right) - w\left(E \setminus \bigcup_{l=k}^{s} j_l\right).
$$

But then

$$
\begin{aligned}
(s-1)w(E) &= \sum_{k=1}^{s-1} w(E) \\
&\geq \sum_{k=1}^{s-1} \left(w(E \setminus \{j_k\}) + w\left(E \setminus \bigcup_{l=k+1}^{s} j_l\right) - w\left(E \setminus \bigcup_{l=k}^{s} j_l\right)\right) \\
&= \sum_{k=1}^{s-1} (w(E \setminus \{j_k\})) + w(E \setminus \{j_s\}) - w\left(E \setminus \bigcup_{l=1}^{s} j_l\right) \\
&= \sum_{k=1}^{s} w(E \setminus \{j_k\}) - w\left(E \setminus \bigcup_{l=1}^{s} j_l\right) \\
&= \sum_{j \in S(i)} w(E \setminus \{j\}) - w(E \setminus S(i)).
\end{aligned}
$$

Since all players in M_S are null players[6] in v, for every $i \in M_S$ it holds that $w(E \setminus \{i\}) = w(E \setminus S(i))$, and thus $(s-1)w(E) \geq \sum_{j \in S(i)} w(E \setminus \{j\}) - w(E \setminus \{i\})$, which is equivalent to

$$
w(E) - w(E \setminus \{i\}) \leq sw(E) - \sum_{j \in S(i)} w(E \setminus \{j\}) = \sum_{j \in S(i)} (w(E) - w(E \setminus \{j\})).
$$

[6]Player $i \in N$ is a ***null player*** in v if $v(E) = v(E \setminus \{i\})$ for all $E \subset N$.

With (2) it then follows that $\varphi_i(f,S) = \varphi_i(v,S) = Sh_i(w) \leq \sum_{j\in S(i)} Sh_j(w) = \sum_{j\in S(i)} \varphi_j(v,S) = \sum_{j\in S(i)} \varphi_j(f,S)$.
Thus the second inequality in Proposition 2.3 is satisfied.

□

PROOF OF PROPOSITION 2.4

Let (N,f,S) be a hierarchically structured firm and let $i \in M_S$. Further, let (N,v,S) be the corresponding game with permission structure as described in Appendix A, and let $w = \mathcal{R}_S(v)$.

(i)

Let $j \in S(i)$ be such that for every $h \in \overline{S}(i) \setminus \overline{S}(j)$ and every $E \subset N$ with $E \cap \overline{S}(j) = \emptyset$ it holds that $f(x^E) = f(x^{E\setminus\{h\}})$. Then

(i) $w(E) - w(E\setminus\{i\}) = 0$ if $E \ni i$, $E \not\ni j$;

(ii) $w(E) - w(E\setminus\{j\}) = 0$ if $E \not\ni i$, $E \ni j$;

(iii) $w(E) = w(E\setminus(\overline{S}(i)))$ for all $E \subset N$ with $E \cap \overline{S}(j) = \emptyset$.

From this and the facts that $w(E\setminus\{i\}) = w(E\setminus\overline{S}(i))$, $w(E\setminus\{j\}) = w(E\setminus\overline{S}(j))$ for all $E \subset N$, and $\overline{S}(j) \subset \overline{S}(i)$ it follows that

$$\begin{aligned}
\varphi_i(f,S) &= \varphi_i(v,S) = Sh_i(w) = \sum_{E\ni i} p(E)(w(E) - w(E\setminus\{i\})) \\
&= \sum_{E\ni i,j} p(E)(w(E) - w(E\setminus\{i\})) \\
&= \sum_{E\ni i,j} p(E)(w(E) - w(E\setminus\overline{S}(i)))
\end{aligned}$$

$$
\begin{aligned}
&= \sum_{E \ni i,j} p(E)(w(E) - w((E \setminus \overline{S}(j)) \setminus \overline{S}(i)) \\
&= \sum_{E \ni i,j} p(E)(w(E) - w(E \setminus \overline{S}(j))) \\
&= \sum_{E \ni i,j} p(E)(w(E) - w(E \setminus \{j\})) \\
&= \sum_{E \ni j} p(E)(w(E) - w(E \setminus \{(j\})) = Sh_j(w) = \varphi_j(v,S) = \varphi_j(f,S).
\end{aligned}
$$

(ii)

Suppose that for every $h \in \overline{S}(i)$ and every $E \ni h$ it holds that $f(x^E) = f(x^{E \setminus \{h\}}) + f(x^{\{h\}})$. Then

(i) $w(E) - w(E \setminus \{i\}) = \sum_{h \in \sigma(E) \cap \overline{S}(i)} v(h)$ for all $E \ni i$;

(ii) $w(E) - w(E \setminus \{j\}) = \sum_{h \in \sigma(E) \cap \overline{S}(j)} v(h)$ for all $j \in S(i)$ and $E \ni j$;

(iii) $\overline{S}(i) \cap \overline{S}(j) = \overline{S}(j)$ for all $j \in S(i)$;

(iv) $w(E) - w(E \setminus \{j\}) = 0$ if $E \not\ni i,\ E \ni j$.

With this it follows that

$$
\begin{aligned}
\varphi_i(f,S) &= \varphi_i(v,S) = Sh_i(w) = \sum_{E \ni i} p(E)(w(E) - w(E \setminus \{i\})) \\
&= \sum_{E \ni i} p(E) \left(\sum_{h \in \sigma(E) \cap \overline{S}(i)} v(\{h\}) \right) \\
&= \sum_{E \ni i} \sum_{j \in S(i) \cap E} p(E) \left(\sum_{h \in \sigma(E) \cap \overline{S}(i) \cap \overline{S}(j)} v(\{h\}) \right)
\end{aligned}
$$

$$
\begin{aligned}
&= \sum_{j\in S(i)} \sum_{E\ni i,j} p(E)\left(\sum_{h\in\sigma(E)\cap\overline{S}(j)} v(\{h\})\right) \\
&= \sum_{j\in S(i)} \sum_{E\ni i,j} p(E)(w(E) - w(E\setminus\{j\})) \\
&= \sum_{j\in S(i)} \sum_{E\ni j} p(E)(w(E) - w(E\setminus\{j\})) \\
&= \sum_{j\in S(i)} Sh_j(w) = \sum_{j\in S(i)} \varphi_j(v,S) = \sum_{j\in S(i)} \varphi_j(f,S).
\end{aligned}
$$

□

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